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Inertial Circulation of the Western Mediterranean Sea (*).

N. PINARDI (**)
Istituto di Fisica dell'Università - Bologna, Italia

A. SPERANZA and A. TREVISAN
FISBAT-CNR, c/o Istituto di Fisica dell'Università - Bologna, Italia

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Summary. — The essential features of the steady circulation of the western Mediterranean are reproduced by means of a vertically integrated model. In the framework of barotropic theory, it is shown how the effects of the geometry of the basin, including the islands, of the variable bottom topography and of the forcing of the flow at the Gibraltar Strait concur in determining the circulation pattern. Some observed features in particular regions, such as the Ligurian Sea, which are not reproduced by the model, indicate where baroclinic and/or wind-driven dynamics are likely to dominate.

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1. – Introduction.

A remarkable amount of information regarding the hydrological properties of Mediterranean water is currently available [1-3], and some interesting physical

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(**) Present affiliation: Center for Earth and Planetary Physics, Harvard University, Cambridge, MA, U.S.A.
(1) G. WÜST: *J. Geophys. Res.*, **66**, 3261 (1961).
(2) H. LACOMBE and P. TCHERNIA: *Caractères hydrologique et circulation des eaux en Mediterranée*, in *The Mediterranean Sea*, edited by R. J. STANLEY (1972), p. 25.
(3) J. P. BETHOUX: *Oceanol. Acta*, **3**, No. 1, 79 (1980).

processes, *e.g.* deep and intermediate water formation ([4-8]) have been observed and modelled in some detail. However, no quantitative theory of the general ocean circulation in the Mediterranean basin has been produced up to now.

The lack of such a theory is particularly felt now that renewed interest seems to be arising as regards the meteo-oceanographical properties of the Mediterranean area.

On the other hand, even a superficial glance at the problem of formulating a theory of general circulation in the Mediterranean Sea reveals that one, probably not the minor, historical reason for the lack of such a theory must be found in the technical difficulties it proposes. The Mediterranean basin is bounded by very complicated lateral contours and it is characterized by an irregular bottom topography; exchanges of both heat and momentum with the atmosphere are quite strong and it is not clear whether thermohaline or wind-driven circulation dominates. Moreover, both could be offset by the circulation forced by the influx-outflux at the Strait of Gibraltar.

In view of such difficulties, it is clear that most of the problems of studying the general circulation of the Mediterranean Sea shall have to be dealt with numerically. In fact, it is with the specific purpose of setting up an adequate numerical model that we have taken the first steps towards reaching a theory of western Mediterranean circulation. We try to make a quantitative assessment of the relative role and importance of different modulating and forcing agents in determining stationary circulations. Given the total lack of knowledge about the system in question, we have made use of vertically integrated equations. After a brief summary of available knowledge on the general circulation in the Mediterranean Sea (sect. **2**), we discuss in sect. **3** the basic equations and their response to the lateral boundaries and, in sect. **4**, the effects induced by bottom topography and forcing by the mouths. Finally, in sect. **5**, we give some tentative conclusions, together with an outline for future work.

2. – The general circulation of the Mediterranean Sea: a summary of observational knowledge.

The Mediterranean Sea is connected with the Atlantic ocean by the Gibraltar Strait and with the Black Sea by the Marmara Sea (see fig. 1). The Strait of Sicily divides the Mediterranean Sea into two basins, western and levantine, which have different dynamical properties.

([4]) MEDOC GROUP: *Nature (London)*, **227**, 5262, 1037 (1970).
([5]) H. STOMMEL, A. D. VOORHIS and D. C. WEBB: *Am. Sci.*, **59**, 6, 716 (1971).
([6]) P. D. KILLWORTH: *Prog. Oceanogr.*, **7**, 2, 59 (1976).
([7]) J. C. GASCARD: *Oceanol. Acta*, **1**, 315 (1978).
([8]) M. CREPON, L. WALD and J. M. NOUGET: *J. Geophys. Res.*, **87**, 595 (1982).

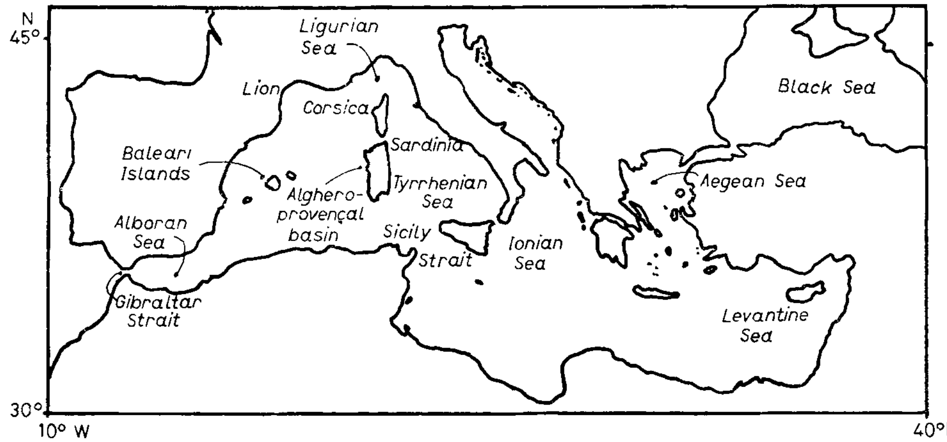

Fig. 1. – A schematic representation of the Mediterranean basin and the subregion denominations.

Its maximum latitudinal extension is approximately 10 degrees around 40° N and 35° N in the western and eastern or levantine basin, respectively; it extends longitudinally from 5° W to 35° E. The total volume of water is about $3.5 \cdot 10^6$ km^3 (9), and the mean residence time is approximately 100 years. The average depth of the whole basin is 100 m and the maximum depth of almost 3500 m is reached in the centre of the Tyrrhenian Sea.

The western basin can be distinguished from the eastern one by the lack of marginal seas such as the Adriatic and Aegean which are present in the eastern part of the Mediterranean. The only significant western compartment is defined by the Sardinian-Corsican islands which separate the Alghero-Provençal basin from the Tyrrhenian Sea. We shall first concentrate on the description of the hydrological and meteorological conditions of the western basin; a more complete description of the circulation features in the whole of the Mediterranean can be found in BETHOUX (3,10) and BUNKER (11).

The basic features of the regional climatology of the basin are warm, dry Summers and cold, dry, intense north-westerly winds during Winter. The evaporation budget exceeds the rainfall and river run-off budget so that the mean annual deficit of water is $0.7 \cdot 10^{12}$ m^3/year (see ref. (3)) in the western basin. The maximum water deficit is attained along the coasts of North Africa and is somewhat less in the Tyrrhenian and Alghero-Provençal basins.

The temperature and salinity profile for the western Mediterranean are

(9) A. R. MILLER and R. J. STANLEY: *Rapp. P-V Reun. Cons. Int. Explor. Mer*, **18**(3), 755 (1965).
(10) J. P. BETHOUX: *Oceanol. Acta*, **2**, No. 2, 218 (1979).
(11) A. F. BUNKER: *J. Phys. Oceanogr.*, **2**, 225 (1972).

shown in fig. 2 for one station in the middle of the Alghero-Provençal basin: the lack of western basin averaged data is conspicuous but we retain that these profiles can serve to describe the properties of the water masses of the basin. The surface water extends to a depth of (100÷200) m; in the western basin, the lower limit of the surface water layer is indicated by a temperature minimum and its salinity characteristics are related to the Atlantic water inflowing into the basin through the Strait of Gibraltar.

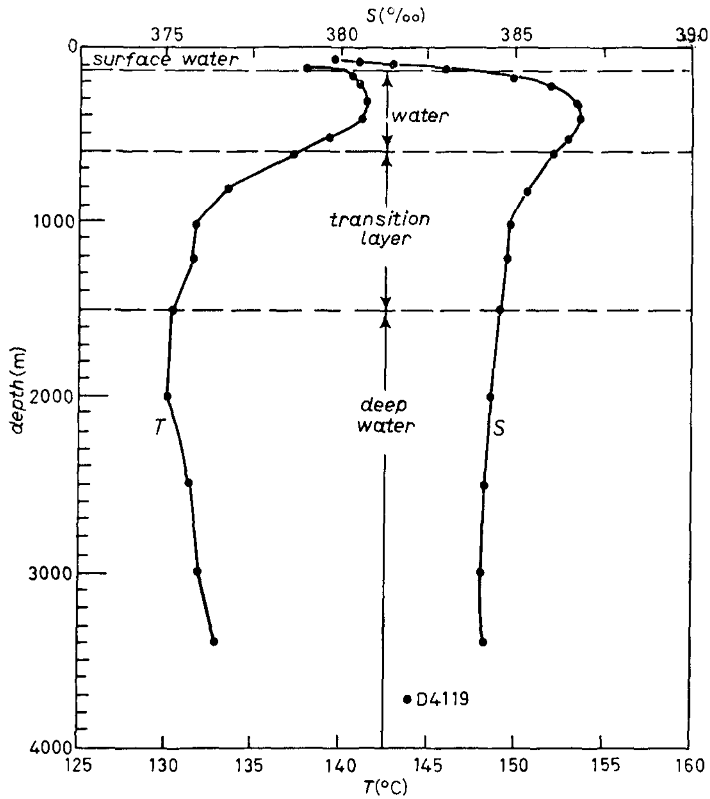

Fig. 2. – Salinity and temperature profiles for the western Mediterranean (from Svendrup ([12])).

The second layer of water is characterized by a maximum of salinity and its origin can be traced back to the very eastern part of the Mediterranean basin. This intermediate or levantine water layer extends approximately down to 600 m but its lower boundary is not as well defined as its upper one, since the slope of the salinity and temperature profiles from 600 m to 1500 m is almost constant.

([12]) H. U. Sverdrup: in *The Oceans* (Prentice-Hall, New York, N. Y., 1972).

The deep water is characterized by a slight increase in temperature and has very different physical properties from those of the eastern basin deep water since the shallow sill of the Strait of Sicily occludes any exchange of deep waters.

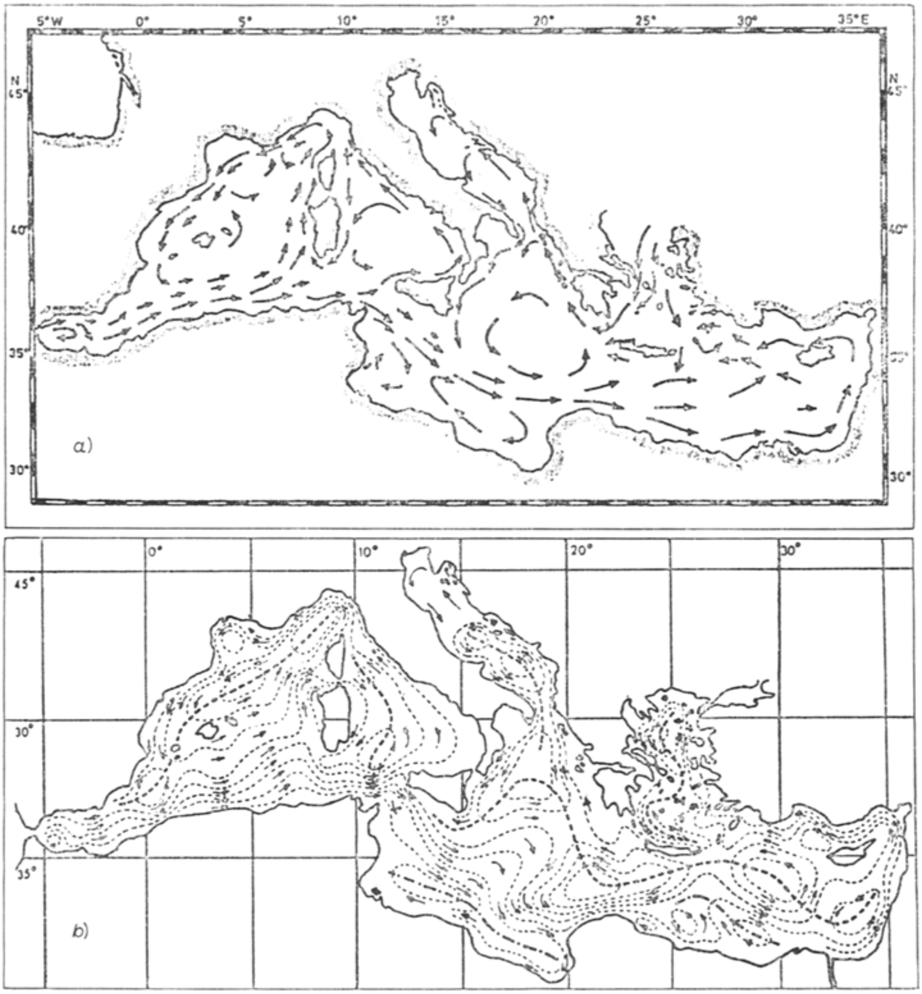

Fig. 3. – *a*) Summer surface circulation (from LACOMBE and TCHERNIA [2]); *b*) Winter. surface circulation [13].

The circulation of the superficial waters is shown in fig. 3*a*) and *b*) for Winter and Summer; the intermediate or levantine water circulation pattern is reproduced in fig. 4 for Summer conditions.

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[13] I. M. OVCHINNIKOV: *Oceanology*, **6**, 48 (1966).

The western basin surface flow shows the same large-scale cyclonic circulation both in Winter and Summer but, in Summer, the circulation seems to be less intense. The Atlantic water, entering at Gibraltar, forms a narrow, intense, southern boundary current which, on reaching the Strait of Sardinia, branches into three directions: one continues towards the Strait of Sicily and the other two turn northward on the western side of the Sardinian and Italian

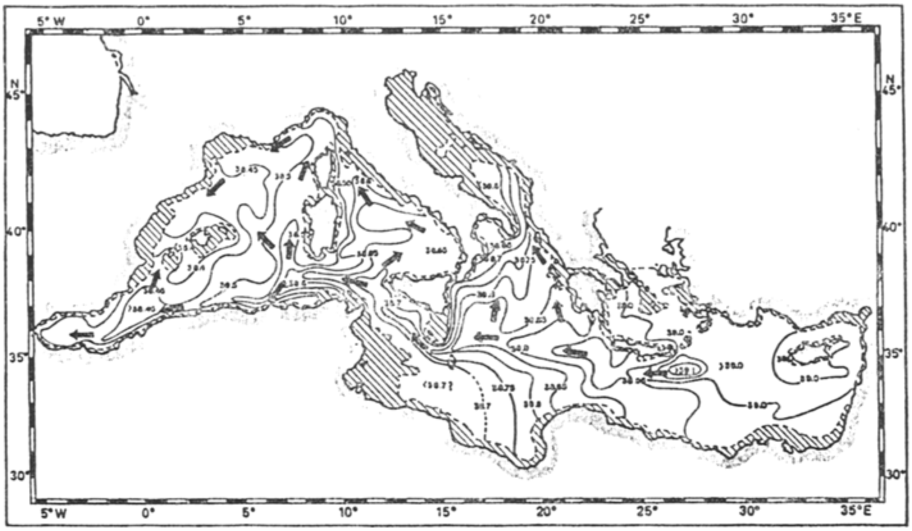

Fig. 4. – Summer intermediate water circulation (1).

coasts, forming a large-scale cyclonic gyre in the Alghero-Provençal basin and in the Tyrrhenian Sea. In the latter, the flow branches successively in a current flowing North, entering the Ligurian-Provençal basin.

Around the islands, the circulation pattern is more complicated and not very well defined; around the Baleari islands, the summer circulation appears to form a close cyclonic gyre while, in Winter, the flow is broader in the centre of the Alghero-Provençal basin. Around Sardinia and Corsica, the circulation is also not very well defined, but it is generally anticyclonic, with large gradients along the western part of the islands.

The intermediate water circulation is not well known in the western basin, except for the branch which enters the Sicily Strait and flows towards the Gibraltar Strait along the northern coasts of Africa. Moreover, the circulation in the Tyrrhenian Sea and Alghero-Provençal basin basically follows the same direction as the surface waters (fig. 4). The bottom circulation is still unknown to our knowledge.

In the Gulf of Lion and in the Ligurian Sea, the phenomenon of deep water formation [4,7,14] occurs during the winter season; as KILLWORTH [15] has pointed out, this process of deep water formation is an open ocean convection mechanism, like the one occurring in the Norwegian and Labrador seas. One of the prerequisites for this occurrence is the existence of a cyclonic circulation and strong cooling at the surface.

3. – Vertically integrated equation for stationary flow and the role of the geometry of the basin.

As we have seen, the vertical stratification of Mediterranean waters is rather complex. Values of the Brünt-Väisälä frequency N range one order of magnitude around $10^{-3}\,\mathrm{s}^{-1}$. Consequently, the slope f_0/N (f_0 is the local value of Coriolis parameter) is of the order of 10^{-1}. The slope of the bottom reaches 1/40 only at selected regions: we do not expect, on the average, topographic generation of nongeostrophic vertical velocities. Since the overall Rossby number is very small, we are justified in trying to model the general circulation of the Mediterranean with the quasi-geostrophic equations. If we consider the average depth to be of the order of 1.5 km, the value of the slope gives an internal Rossby radius of deformation of the order of 15 km.

We start from horizontal momentum and continuity equations for a homogeneous, incompressible fluid in stationary flow at the first order of expansion in the Rossby number:

$$u^{(0)}\frac{\partial u^{(0)}}{\partial x}+v^{(0)}\frac{\partial u^{(0)}}{\partial y}-fv^{(1)}=-\frac{1}{\varrho^{(0)}}\frac{\partial p^{(1)}}{\partial x}\,,\tag{3.1}$$

$$u^{(0)}\frac{\partial v^{(0)}}{\partial x}+v^{(0)}\frac{\partial v^{(0)}}{\partial y}+fu^{(1)}=-\frac{1}{\varrho^{(0)}}\frac{\partial p^{(1)}}{\partial y}\,,\tag{3.2}$$

$$\frac{\partial u^{(1)}}{\partial x}+\frac{\partial v^{(1)}}{\partial y}+\frac{\partial w^{(1)}}{\partial z}=0\,,\tag{3.3}$$

where u, v and w are the respective zonal latitudinal and vertical velocity component, p is the pressure, ϱ_0, a reference density and f, the Coriolis parameter. Consider the derived vorticity equation

$$\boldsymbol{v}^{(0)}\cdot\nabla\zeta^{(0)}=-f\nabla\cdot\boldsymbol{v}^{(1)}=f\,\frac{\partial w^{(1)}}{\partial z}\,,\qquad \text{where } \zeta=-\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\,.\tag{3.4}$$

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(14) R. STOMMEL: *Deep winter-time convection in the western Mediterranean Sea*, in *Studies in Physical Oceanography: a Tribute to Georg Wüst on his 80 birthday*, Vol. 2, edited by A. L. GORDON (Gordon and Breach, New York, N. Y., 1972), p. 207.
(15) P. D. KILLWORTH: *Rev. Geophys. Space Phys.*, **21**, 1 (1983).

Vertical integration of (3.4) between the bottom $z = H - h(x, y)$ and the top $z = \eta(x, y)$ gives

$$\int_{H-h}^{\eta} \mathbf{v}^{(0)} \cdot \nabla\zeta^{(0)}\, \mathrm{d}z = f[w^{(1)}]_{H-h}^{\eta} . \tag{3.5}$$

We assume the top surface to be a rigid lid

$$\eta(x, y) = 0 , \qquad w^{(1)}(0) = 0 .$$

The bottom slope enters (3.5) at the first order in the Rossby number expansion, so that $W^{(1)} = \mathbf{v}^{(0)} \cdot \nabla h$ at $z = H - h(x, y)$.

The explicit form of (3.5) is now

$$\int_{H-g}^{0} \mathbf{v}^{(0)} \cdot \nabla\zeta^{(0)}\, \mathrm{d}z = -f(\mathbf{v}^{(0)})_{z=H-h} \cdot \nabla h . \tag{3.6}$$

The average vertical value of $\mathbf{v}^{(0)}$ obviously differs from its bottom value. However, in modelling average orientation, we can circumvent this difficulty by noticing that the vertical profiles are never very far from a typical shape and this allows us to assume a fixed ratio between $\langle \mathbf{v}^{(0)} \rangle$ and $(\mathbf{v}^{(0)})_{z=H-h}$. We assume such a ratio to be order unity and obtain

$$(H - h)\langle \mathbf{v}^{(0)} \rangle \cdot \nabla\zeta^{(0)} + f\langle \mathbf{v}^{(0)} \rangle \cdot \nabla h = 0 , \tag{3.7}$$

where H is a representative depth of the basin and $\langle\ \rangle$ represents the vertical average.

At this point, by introducing the quasi-geostrophic stream function $\psi = (p^{(0)} - p_s)/(\varrho_0 f_0)$ ($p_s(z)$ is the standard pressure at the depth z, f_0 the local value of the Coriolis parameter), we can finally write eq. (3.7) in its well-known form

$$J\left(\psi, \frac{\zeta^{(0)} + f}{H - h}\right) = 0 . \tag{3.8}$$

Since we wish, in this section, to explore the role of lateral boundaries, we assume $h = \text{const}$. By introducing the β-plane approximation into (3.8), we obtain

$$J(\psi, \nabla^2\psi + \beta(y - y_0)) = 0 , \tag{3.9}$$

where y_0 is the latitude around which the Coriolis parameter is expanded (in the case of the western Mediterranean $y_0 = 40°\,\mathrm{N}$). Equation (3.9) is the classical equation for inertial flow (see FOFONOFF [16]). The usual boundary condition is $\psi = \text{const}$ at lateral contours. This equation is known to possess an infinite number of solutions: any solution of the equation

$$\nabla^2\psi + \beta(y - y_0) = F(\psi)\,, \tag{3.10}$$

where $F(\psi)$ is a generic functional of ψ, will satisfy (3.9). Equation (3.10) can be solved for different linear and nonlinear forms of $F(\psi)$. Nonlinear functionals have been discussed in some detail in the literature. However, this aspect of the problem is not our specific interest here. The introduction of noninertial forces eliminates the ambiguity in the determination of $F(\psi)$ which, for a limited excursion of the stream function, can be expanded in powers and linearized:

$$\nabla^2\psi + \beta(y - y_0) = F(\psi) = C_0 + C_1\psi\,. \tag{3.11}$$

In order to determine completely the differential problem (3.11), we have to specify the constants C_0 and C_1. The constant C_1 is easily determined once we represent the stream function as the sum of a uniform zonal flow plus a part possessing vorticity $\psi = Uy + \psi'$. Substitution into (3.11) gives

$$\nabla^2\psi' + \beta y - \beta y_0 = C_0 + C_1 Uy + C_1\psi'\,. \tag{3.12}$$

It is clear that the condition for cancellation of the y-dependent part is that $C_1 = \beta/U$. C_1 can, therefore, be connected with an overall zonal flow which, however, is arbitrary in our context. Determining C_0 is more problematic. It is clear from (3.10) that this constant represents an overall uniform vorticity of the system. We can, therefore, calculate it from the integral of the vorticity equation.

When $C_1 = \beta/U$ is inserted into (3.12), it reduces to

$$\nabla^2\psi' = \beta y_0 + C_0 + \frac{\beta}{U}\psi'\,. \tag{3.13}$$

If we define M as the total relative vorticity, by integration of (3.13) over the area S of the basin, we obtain

$$M = \int_S \nabla^2\psi'\,\mathrm{d}s = (\beta y_0 + C_0)S + \frac{\beta}{U}\int \psi'\,\mathrm{d}s\,. \tag{3.14}$$

If the scale $\sqrt{U/\beta}$ is much smaller than the scale of the basin we have to deal with boundary layers of thickness $\sqrt{U/\beta}$ as clear from (3.13).

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[16] N. P. FOFONOFF: *J. Mar. Res.*, **13**, 254 (1954).

Figures 5 and 6 show the stream function obtained by imposing, respectively, the boundary layer at the southern and northern boundary. This can be done by choosing appropriate values for C_0. These two cases correspond to overall relative vorticities of opposite signs.

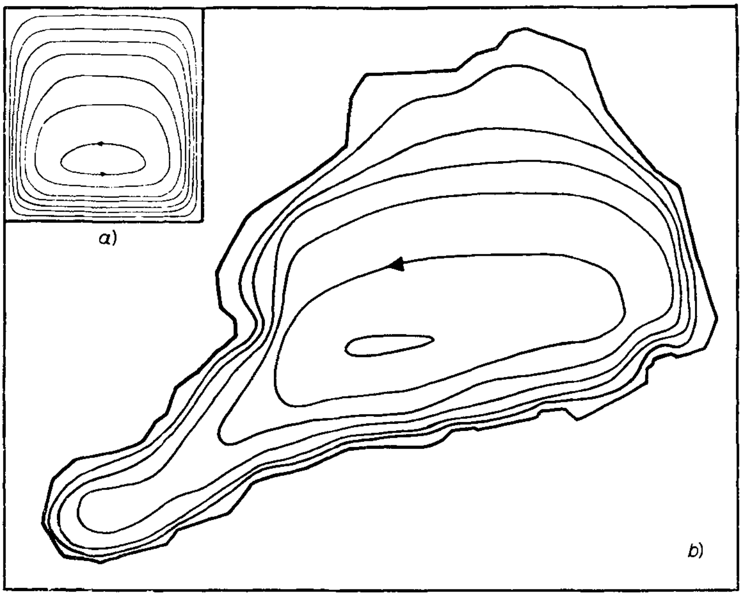

Fig. 5. – *a*) Solution of eq. (3.10) with $C_0 = 0$ (arbitrary units); *b*) the same as in *a*), but for the Mediterranean basin geometry (arbitrary units).

The boundary condition is $\psi = 0$ along the coastlines: the parameters used in the numerical integration scheme, together with the grid spacing, are given in appendix A. Figures 5 and 6 show the stream function pattern in a square and in a closed domain with the shape of the western Mediterranean, for the two different above choices of C_0. The jetlike zonal flow is seen to move from the southern (fig. 5) to the northern (fig. 6) boundary. From a physical point of view, the solution considered here simulates the stationary solution of the barotropic problem with forcing and dissipation, characterized by the same U and M. It is, however, important to notice that the symmetry properties of ψ do not allow the representation of East-West asymmetric features of the circulation like, for example, western boundary currents. In order to represent such asymmetric features, explicit insertion of wind and thermal forcing is required.

An immediate improvement of the model circulation pattern, with respect to the real circulation (fig. 3*a*)), is obtained by introducing the islands into the geometry of the basin. In fig. 7*a*) and *b*), the solution of (3.10) is displayed for $C_0 = 0$ and $\psi = 0$ along all the boundary contours for different values of U.

The islands of Sardinia and Corsica now force the steady inertial circulation to articulate in a series of sub-basin gyres and in the case of $U = 5\ \mathrm{cm\ s^{-1}}$ (fig. 7*b*)), a small closed circulation forms in the southern part of the Tyrrhenian basin. In the Alghero-Provençal basin, the flow, although significantly distorted with respect to the case without islands, is still zonal. In the case

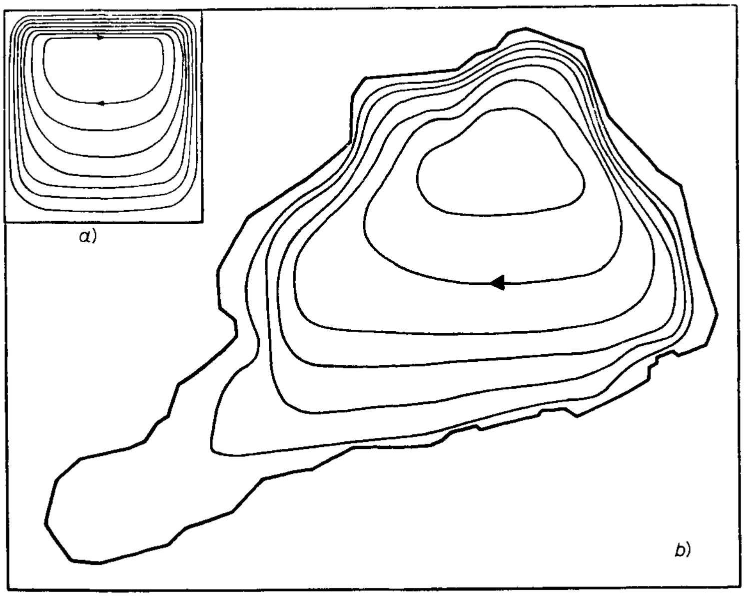

Fig. 6. – *a*) Solution of eq. (3.10) with $C_0 = -\beta_0 y_0$ (arbitrary units); *b*) the same as in *a*), but for the Mediterranean basin geometry (arbitrary units).

of fig. 7*a*), a wide, along-shore basin circulation is present, embedding the smaller scale cyclonic circulations within the subregions of the domain. The southern boundary current is remarkably realistic.

The value of ψ at the boundaries is arbitrary in our problem. The imposition of $\psi = 0$ at the boundaries of the domain is a strong unphysical constraint, especially because it implies no net transport between the islands and the continental coastline. We have thus introduced a more realistic distribution of the transport between subregions of the domain and inflow-outflow conditions at the Straits of Gibraltar and Sicily. In appendix B, the exact values of the stream function along the boundary contours are given for the values of the transports used. The resulting circulation is shown in fig. 8: the introduction of the forcing at the Strait of Gibraltar and Sicily does not drastically modify the characteristics of the flow with respect to the case in fig. 7*a*). The constants giving rise to the definition of the two sets of boundary conditions are derived from the phenomenological evaluations of fluxes T_1, T_2 and T_3 in one case and T_1, T_2 and T_4 in the other. The variations caused by these

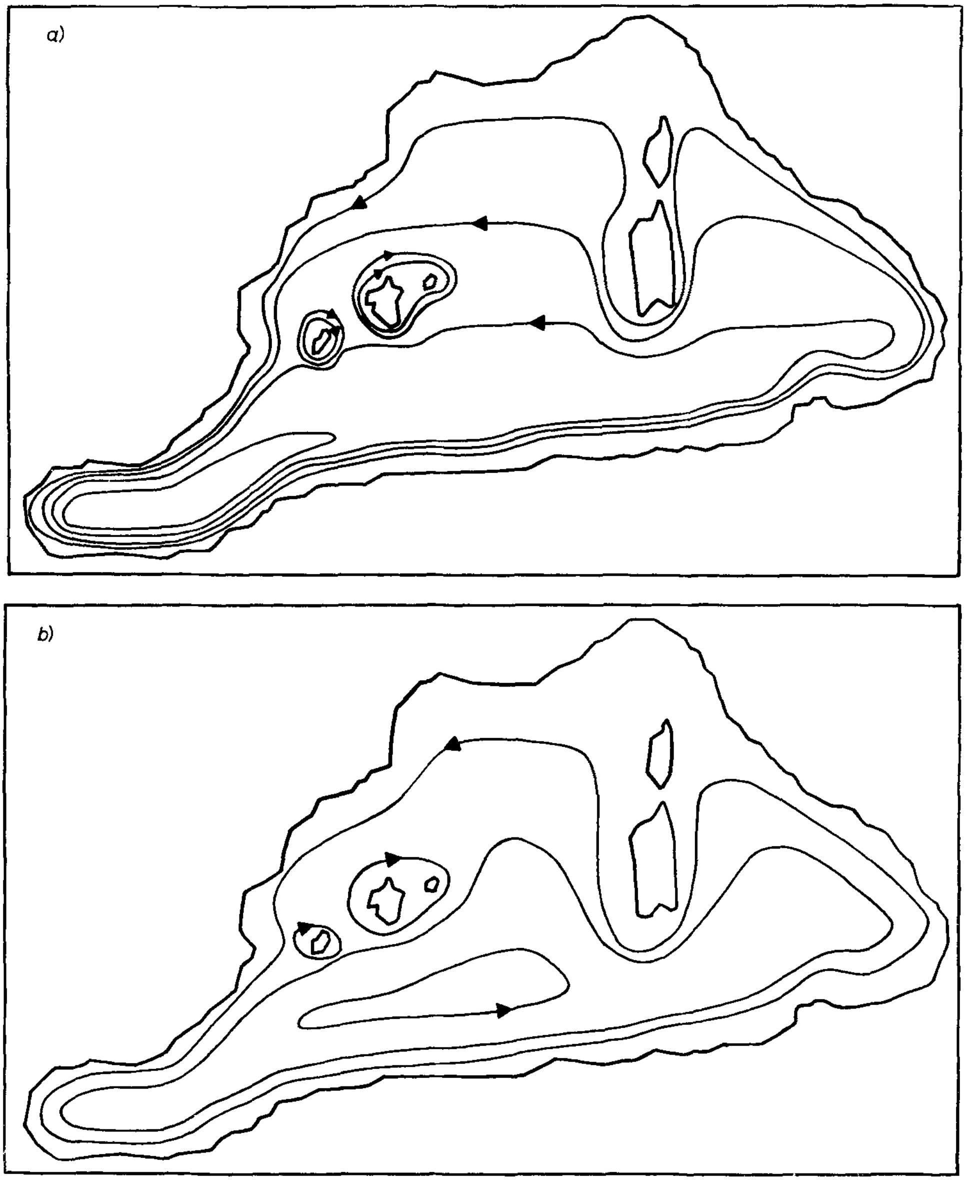

Fig. 7. – Solution of eq. (3.10) with $C_0 = 0$ and $\psi = 0$ at the boundaries of the islands and coastlines: *a*) $U = 0.5$ cm/s ($\Delta\psi = 0.5$ m/s); *b*) $U = 5$ cm/s ($\Delta\psi = 5$ m/s).

two different but physically consistent sets of boundary conditions do not modify the nature of the solutions.

In the next section, we shall see that the introduction of a dynamical effect such as bottom topography is responsible for a great improvement in the realism of the model.

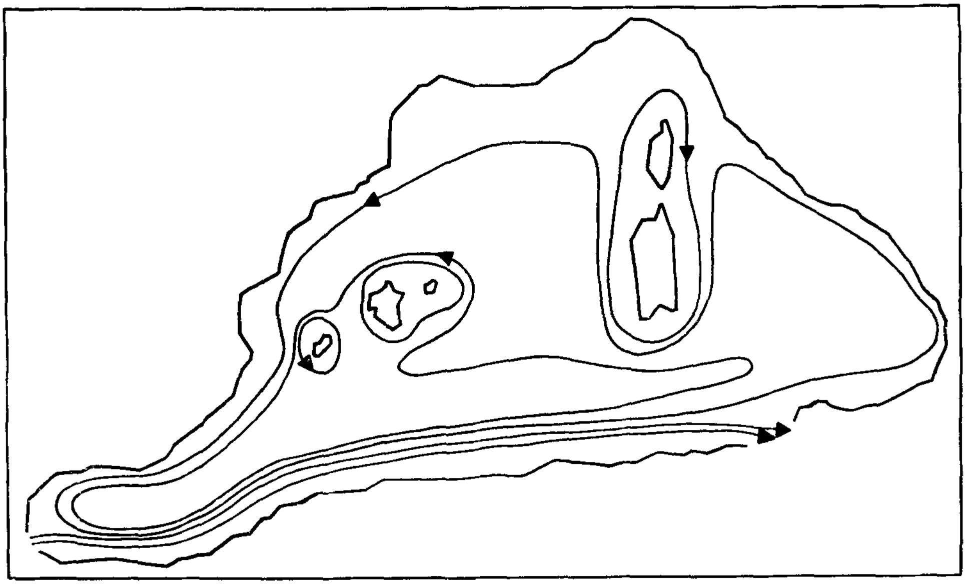

Fig. 8. – Solution of eq. (3.10) with $C_0 = 0$, but for the boundary conditions (B.5) and $U = 0.5$ cm/s ($\Delta\psi = 1$ m/s).

4. – The effect of bottom topography and inflow-outflow at the Straits.

In order to study the effect of bottom topography, we have to go back to (3.8) which, as we have seen in the preceding section, can be equivalently written as

$$\frac{\nabla^2 \psi + f_0 + \beta(y - y_0)}{1 - h/H} = F(\psi) . \tag{4.1}$$

In fig. 9, the topography used in eq. (4.1) is shown: the maximum depth reached in the western Mediterranean basin (South Tyrrenian) is 3477 m. Choosing an average bottom depth $H = 1750$ m, the excursion of h is not small. Therefore, we do not expand the denominator of (4.1) in powers of h/N as is usually done. The quasi-geostrophic approximation used to obtain eq. (4.1) remains, however, consistent since the vertical velocity forced by the gradient of the topography is still considerably smaller than the horizontal velocity.

Assuming a linear function $F(\psi)$, with $C_0 = 0$, we obtain the following equation:

$$\nabla^2 \psi - \frac{\beta}{U}\left(1 - \frac{h}{H}\right)\psi = -\beta(y - y_0) - \frac{f_0 h}{H} . \tag{4.2}$$

This equation has been numerically integrated, for different values of U, with

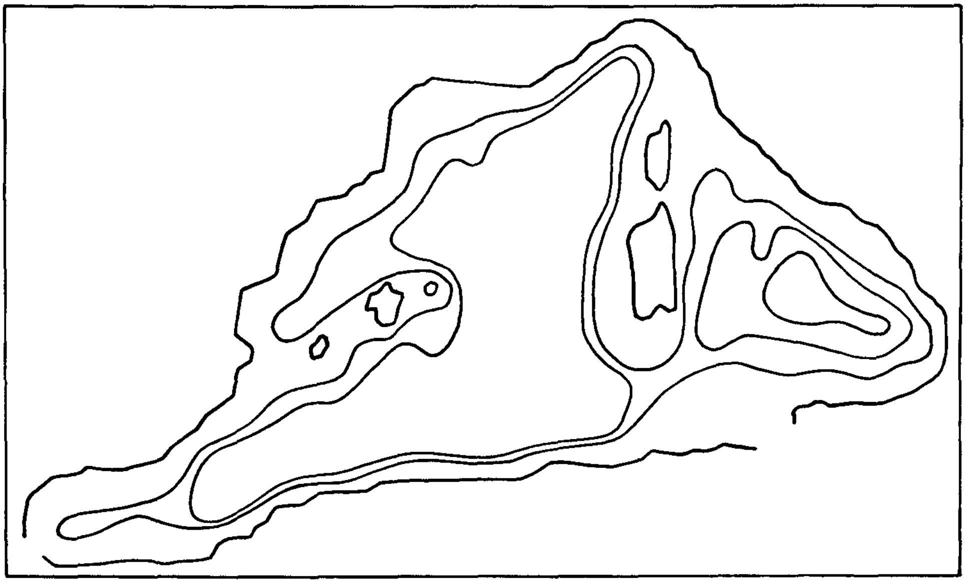

Fig. 9. - Bottom topography of the basin. Contour interval 300 fathoms.

the boundary conditions along the lateral contours of the basin discussed in the previous section and the bottom topography shown in fig. 9. The results are shown in fig. 10*a*) and *b*): they should be compared with fig. 7*a*) and *b*) which correspond to the same choice of boundary conditions and U values.

The introduction of the topography has the important effect of inducing closed circulation in the major sub-basins; furthermore, the current system along the coasts becomes stronger and assumes the connotation of a strong filamenting flow, branching at various locations in a very similar way to the pattern in fig. 3*a*). Also noticeable are the formation of the cyclonic gyre in the Tyrrenian, the anticyclonic gyre in the Alborean Sea and the well-defined northward flow along the western side of the islands of Sardinia and Corsica. Moreover, the anticyclonic circulation in the Gulf of Lion is not seen in reality and the circulation around the Baleari islands seems to be in the opposite direction to the one in fig. 3*a*). The lack of wind and thermohaline forcing could be responsible for the misrepresentation of these features.

In order to study the combined effects of influx/outflux at the mouths of the western Mediterranean basin and topography, we can make use of the simple scheme used previously which is described in appendix B.

The circulation obtained by integrating (4.2), using boundary condition (B.3), is shown in fig. 11. The comparison with fig. 10*a*) shows that the forcing at the Straits of Gibraltar and Sicily induces a realistic basin-wide circulation without modifying the characteristics of the circulation captured with the introduction of the topography.

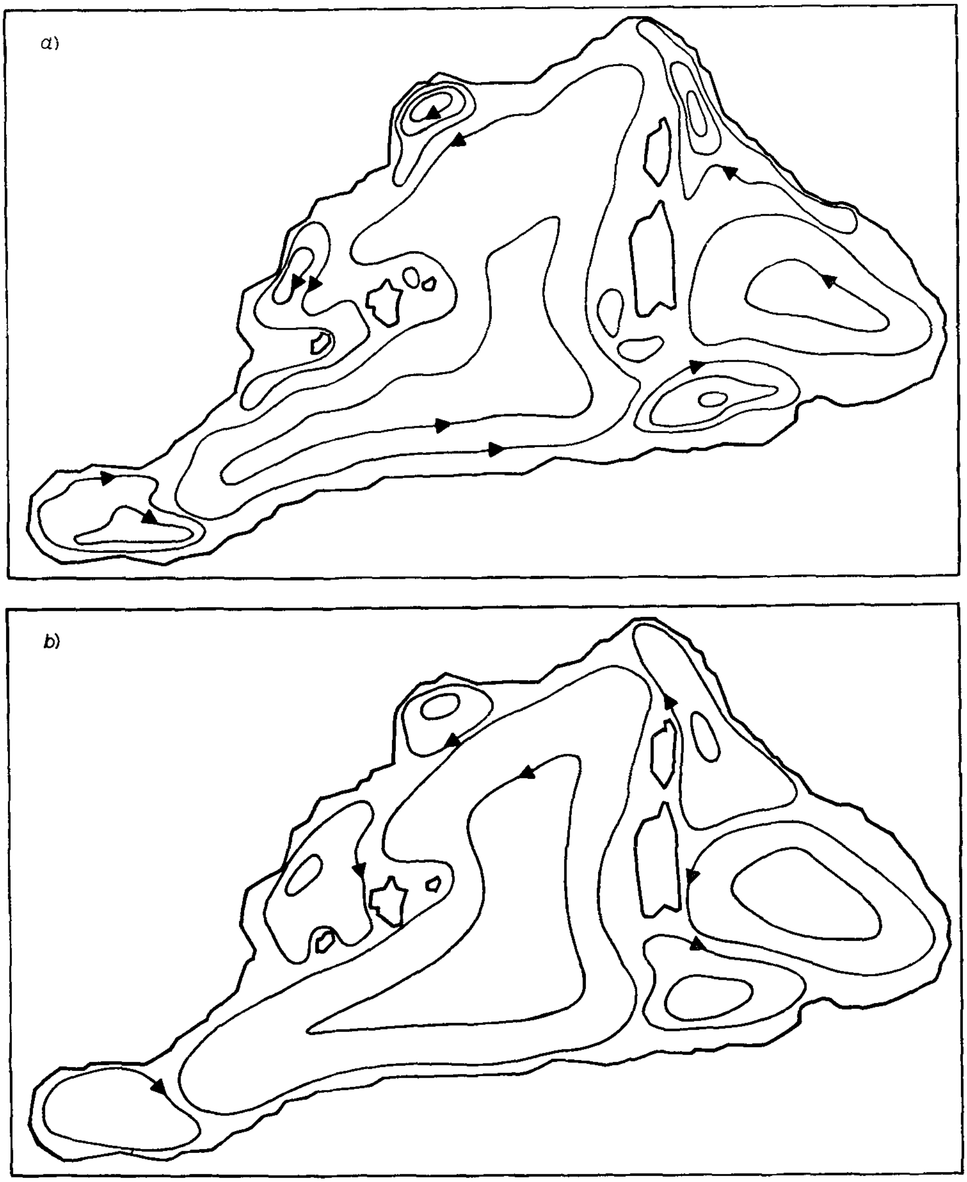

Fig. 10. – Solutions of eq. (4.2) with $\psi = 0$ along all the boundary contours: *a*) $U = 0.5$ cm/s ($\Delta\psi = 10$ m/s); *b*) $U = 5$ cm/s ($\Delta\psi = 50$ m/s).

The topography, together with influx/outflux at the straits, induces a highly realistic southern boundary current which branches at the Sardinian Strait towards the North along the contours of constant ambient potential vorticity. The gyre in the Tyrrhenian Sea seems to be primarily due to the topographic effects since the inclusion of topography in the model leads to a realistic gyre. Moreover, the topographically induced circulation enhances a

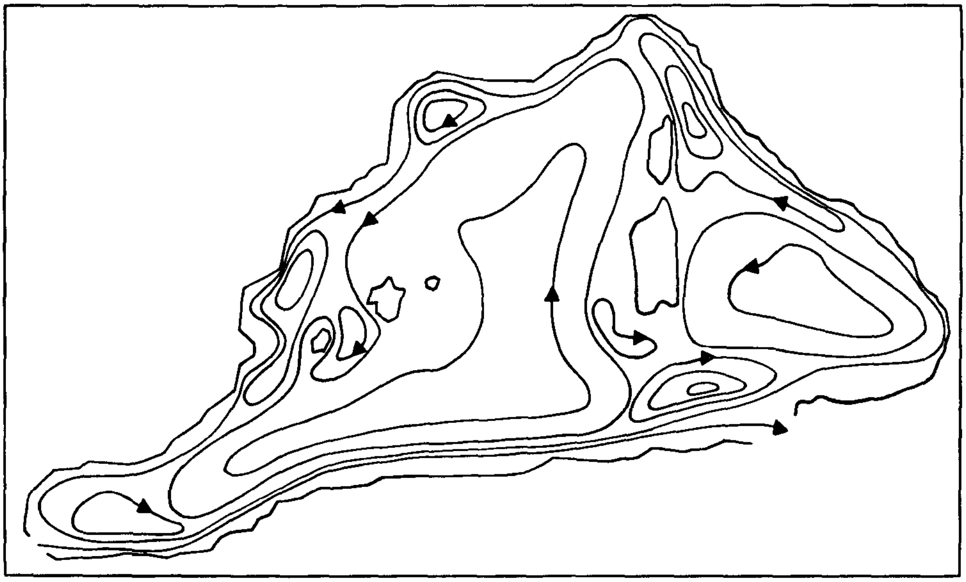

Fig. 11. – Solution of eq. (4.2) with boundary conditions (B.3) with $U = 0.5$ cm/s ($\Delta\psi = 10$ m/s).

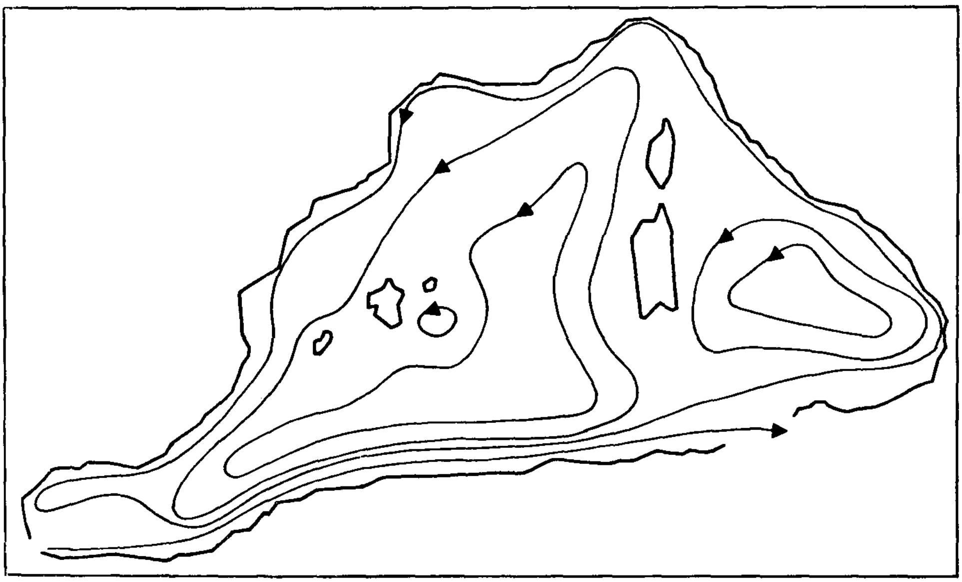

Fig. 12. – Solution of eq. (4.2) with boundary conditions (B.5) and $U = 0.5$ cm/s ($\Delta\psi = 10$ m/s).

circulation around the Baleari which is not observed in the maps of fig. 3*a*) and *b*). We have already pointed out that, in this region, the dynamical balance described by the model is probably inadequate.

In fig. 12, other boundary conditions (see (B.5)) have been imposed which are the same as those in fig. 8. The comparison between fig. 12 and 8 shows that the addition of the topography actually improves the realism of the circulation.

A qualitative comparison of our results with the observed circulations leaves us with the impression that the essential features are captured. A significant inadequacy is the absence of cyclonic vortices in the Ligurian-Provençal area. A possible explanation is that such features are essentially baroclinic (see KILLWORTH [6]). However, it is a fascinating, although at this stage, purely speculative hypothesis, that the failure to represent correctly the average circulation in the Ligurian-Provençal region may be due to the lack of western boundary intensification effects in the inertial theory. If this is the case, forcing due to North-westerly winds modifies the inertial current, producing the observed countercurrents. The testing of this hypothesis is one of the primary objectives of future research.

5. – Conclusions.

Given the very preliminary nature of our study and the drastic simplifications to which we have subjected the dynamic equations in order to make the mathematics legible, even in the complicated geometry of the Mediterranean area, our conclusions are necessarily tentative.

The impression that we receive from comparison among the different patterns of stationary circulation obtained and with observed circulations, is that lateral contour, bottom topography and inflow/outflow at the mouths all play an equally important role in the circulation of the western Mediterranean Sea. However, we can distinguish the effects of the contemplated steady forcing mechanisms in producing the obtained circulation pattern.

The choice of an integrated relative vorticity in the case of pure inertial circulation provokes the formation of an intense southern boundary current.

The introduction of the topography modifies the former picture introducing limited-area closed circulations. The steady forcing at the mouths induces a basin-wide circulation, improving the degree of realism of the results.

An interesting feature captured by the model is the Alborean Sea anticyclonic gyre, mainly induced by the topography. Furthermore, it should be stressed that the intense current at the southern boundary of the western basin can be simply explained as a component of the steady inertial circulation as discussed in sect. **3**.

The introduction of a «steady forcing», such as bottom topography of the basin, enhances the realism of the model: we believe this is an important dynamical constraint to the flow which should be included in future work which will possibly consider unsteady circulations.

Of course, we know from observations that the vertical structure of the different fields is very complex and the dynamical effects of boundary conditions cannot be completely captured by means of a vertically integrated model. However, the partial success of the barotropic model is not a mere coincidence and it is not surprising to those who have experience in simulation by barotropic models: the equilibrium mean flow in baroclinic ocean models is very similar to the barotropic one which is a solution of eq. (3.10) (see SALMON ([17])). It is also interesting to notice that the constants of the linear functional of (3.10) can be interpreted in the context of statistical theory of geostrophic turbulence as Lagrange multipliers of energy and potential enstrophy ([18]).

However, as already mentioned in the text, severe limitations stem from the inability of the inertial model to reproduce asymmetries resulting from inhomogeneous external forcing. The inability to reproduce gyres observed in the circulation in the Ligurian-Provençal sea could be a manifestation of such a limitation. Therefore, it is our feeling that insertion of forcing and dissipation is the most promising direction in which to orientate future work.

APPENDIX A

Equations (3.10) and (4.2) were numerically integrated in a regularly spaced grid: the method of over-relaxation was used, with an over-relaxation coefficient of 1.87. This grid spacing was chosen to be 15′ in both directions, corresponding to an average grid spacing of 25 km.

The Coriolis parameter has been expanded in series around the latitude of 44° 30′ N and $\beta_0 = 1.63\cdot 10^{-8}\,\mathrm{km}^{-1}\,\mathrm{s}^{-1}$ results. The wave-length of the inertial motion is $\lambda = 2\pi\sqrt{U/\beta_0}$ and, if we take as a typical length scale $L = \sqrt{U/\beta_0}$, the Rossby number is $2.8\cdot 10^3$ for $U = 0.5$ cm/s.

APPENDIX B

Here, we consider the problem of specifying ψ along the contours of the basin as schematically drawn in fig. 13. T_1, T_2, T_3, and T_4 are considered as known transports across the meridional sections marked in fig. 13: their values are listed below in (B.6). T_3 and T_4 were estimated to be $T_3 \simeq 0.5T_2$ and

([17]) R. SALMON: *J. Phys. Oceanogr.*, **12**, 1458 (1982).

([18]) R. SALMON: *Geostrophic turbulence*, in *Proc. S.I.F.*, Course XXX (1982), p. 30.

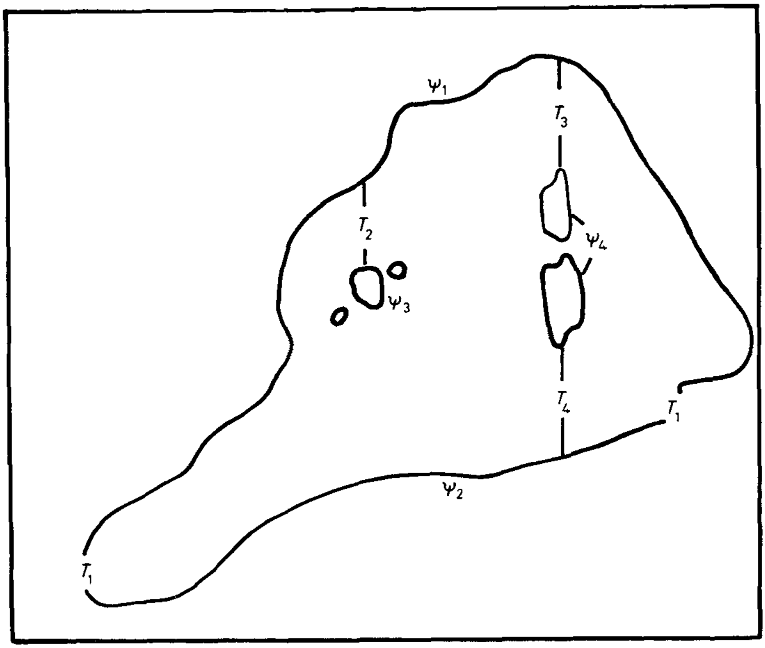

Fig. 13.

$T_4 \simeq (5/8)\,T_1$. The zonal transport across a vertical cross-section Δz is defined to be

$$M_y = -\int_{\Delta z} \psi_y \,\mathrm{d}z \tag{B.1}$$

and, since ψ is a constant in z, it is possible to write

$$M = -\,(\psi(y_1) - \psi(y_2))\,\Delta z = -\,\Delta\psi\,\Delta z$$

(y_1 and y_2 are the maximum and minimum latitudes across which the section of depth Δz extends). In the nomenclature of fig. 13, $\Delta\psi = \psi_1 - \psi_3$ between the Baleari Islands and the coasts of Spain, $\Delta\psi = \psi_1 - \psi_2$ at the Gibraltar and Sicily Straits, $\Delta\psi = \psi_1 - \psi_4$ between the Ligurian coasts and Corsica, and $\Delta\psi = \psi_4 - \psi_2$ at the Sardinia Strait.

Choosing $z = 150$ m (the average depth of the surface layer of Atlantic water at Gibraltar), the system to be solved becomes

$$\left\{\begin{aligned} \psi_1 - \psi_3 &= \,T_2/\Delta z = \,1.2\cdot 10^{-2}\,\mathrm{km^2/s}\,,\\ \psi_1 - \psi_2 &= -\,T_1/\Delta z = -\,0.8\cdot 10^{-2}\,\mathrm{km^2/s}\,,\\ \psi_1 - \psi_4 &= \,T_3/\Delta z = \,0.6\cdot 10^{-2}\,\mathrm{km^2/s}\,. \end{aligned}\right. \tag{B.2}$$

The choice of $\psi_1 = 1.2 \cdot 10^{-2}$ gives the following values for ψ at the boundaries:

$$\psi_2 = 2 \cdot 10^{-2}\ \text{km}^2/\text{s}\,, \qquad \psi_3 = 0.0\ \text{km}^2/\text{s}\,, \qquad \psi_4 = 0.6 \cdot 10^{-2}\ \text{km}^2/\text{s}\,. \tag{B.3}$$

For these values of ψ, the solutions of eq. (4.2) are displayed in fig. 11.

If, instead, we use the system

$$\left\{\begin{aligned} &\psi_1 - \psi_2 = -\,0.8 \cdot 10^{-2}\ \text{km}^2/\text{s}\,,\\ &\psi_1 - \psi_3 = 1.2 \cdot 10^{-2}\ \text{km}^2/\text{s}\,,\\ &\psi_4 - \psi_2 = -\,T_4/\Delta z = -\,0.5 \cdot 10^{-2}\ \text{km}^2/\text{s}\,, \end{aligned}\right. \tag{B.4}$$

with the choice of $\psi_1 = T_2/2\,\Delta z = 0.6 \cdot 10^{-2}$, the boundary conditions become

$$\psi_2 = 1.4 \cdot 10^{-2}\ \text{km}^2/\text{s}\,, \quad \psi_3 = -\,0.6 \cdot 10^{-2}\ \text{km}^2/\text{s}\,, \quad \psi_4 = 0.9 \cdot 10^{-2}\ \text{km}^2/\text{s}. \tag{B.5}$$

With these conditions, the solutions of eqs. (3.10) and (4.2) are displayed in fig. 8 and 12:

$$\left\{\begin{aligned} &T_1 = 1.2 \cdot 10^6\ \text{m}^3/\text{s}\ (^2)\,, \qquad && T_2 = 1.8\ \cdot 10^6\ \text{m}^3/\text{s}\ (^2)\,,\\ &T_3 = 0.9 \cdot 10^6\ \text{m}^3/\text{s}\,, && T_4 = 0.75 \cdot 10^6\ \text{m}^3/\text{s}\,. \end{aligned}\right. \tag{B.6}$$

● RIASSUNTO

Le principali caratteristiche della circolazione stazionaria del Mediterraneo occidentale sono riprodotte tramite un modello integrato verticalmente. Si mostra come, nell'ambito della teoria barotropica, gli effetti della geometria del bacino, inclusa quella delle isole, di una topografia di fondo variabile e della forzatura del flusso allo stretto di Gibilterra concorrono a determinare i vari aspetti della circolazione. Alcuni aspetti locali della circolazione che non sono riprodotti dal modello in particolari regioni, come il Mar Ligure, sono indicativi di dove la dinamica baroclina e/o la forzatura del vento sono da considerarsi dominanti.

Инерциальная циркуляция в западной части Средиземного моря.

Резюме (*). — Существенные особенности устойчивой циркуляции в западной части Средиземного моря могут быть воспроизведены с помощью вертикально проинтегрированной модели. В рамках баротропной теории показывается, как эффекты, связанные с геометрией бассейна, включая острова, и изменяемой топографией дна и силой течния в Гибралтарском проливе, влияют на определение картины циркляции. Некоторые наблюдаемые особенности в отдельных областях, таких как Лигурийское море, которые не могут быть воспроизведены с помощью этой модели, указывают на то, что, по-видимому, диминирует бароклиинная и/или ветряная динамика.

(*) *Переведено редакцией.*